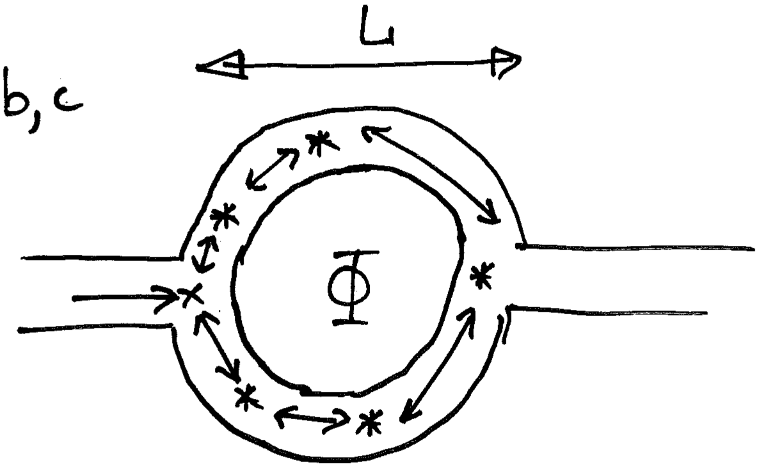
Answers Exam Mesoscopic Physics
8-7-2015

Ⓐ

1 a,b,c



<u>Type 1</u>: time reversed trajectories

AB phase : $\Delta\varphi_{cw} = 2\pi \dfrac{\Phi}{\Phi_0}$ (clockwise)

$\Phi_0 = h/e$ $\quad\quad \Delta\varphi_{ACW} = -2\pi \dfrac{\Phi}{\Phi_0}$ (anticlock wise)

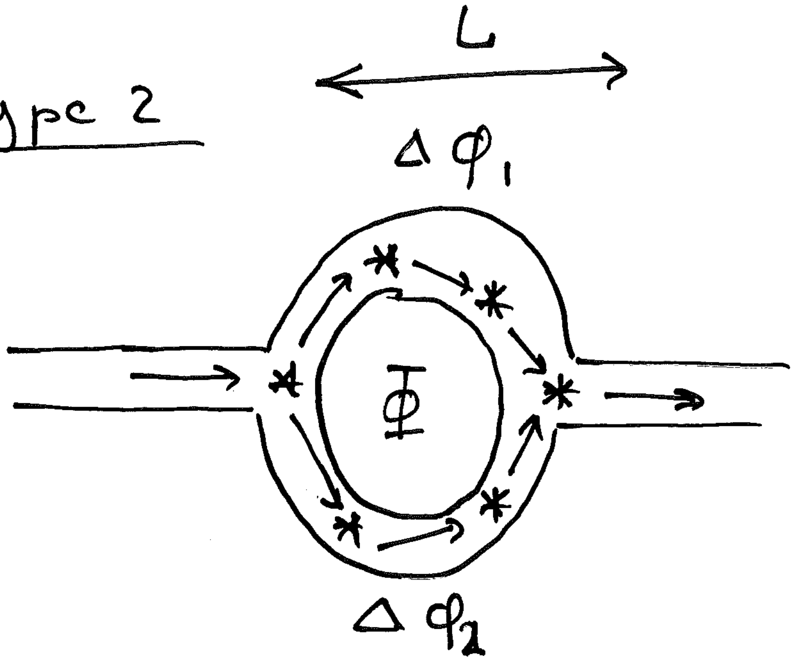
periodicity $= \dfrac{h}{2e}$

required : $L < l_\varphi$

voltage and temperature should be such that $L < l_\varphi$

Type 2

B



AB phase : $\Delta\varphi_1 - \Delta\varphi_2 = 2\pi \dfrac{\Phi}{\Phi_0}$

interference at output depends on

$\Delta\varphi_1 - \Delta\varphi_2 \sim 2\pi \dfrac{\Phi}{\Phi_0}$

periodicity = $\Phi_0 = h/e$

required : $L < l_\varphi$

$kT < E_{th}$

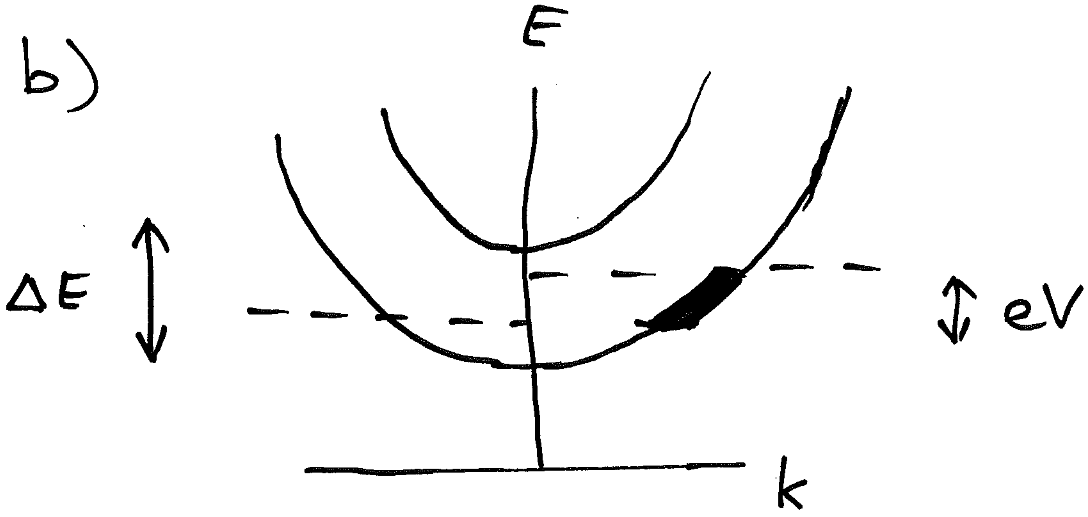
$eV < E_{th}$

$E_{th} = \dfrac{\hbar D}{L^2}$ Thouless energy

C

2 a) see lecture notes

b)



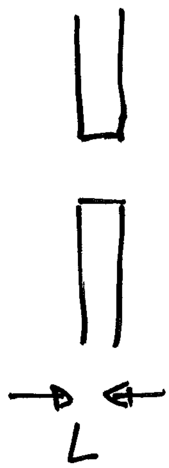
current carried by states in energy interval eV

required: no additional states occupied/empty in other subbands

$\rightarrow$ $eV < \Delta E$

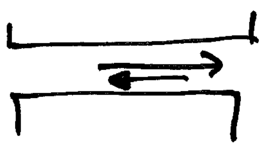
$kT < \Delta E$

2 c) <u>long/short channel</u>



quantum tunneling possible when L is too short

<u>abrupt channel</u>



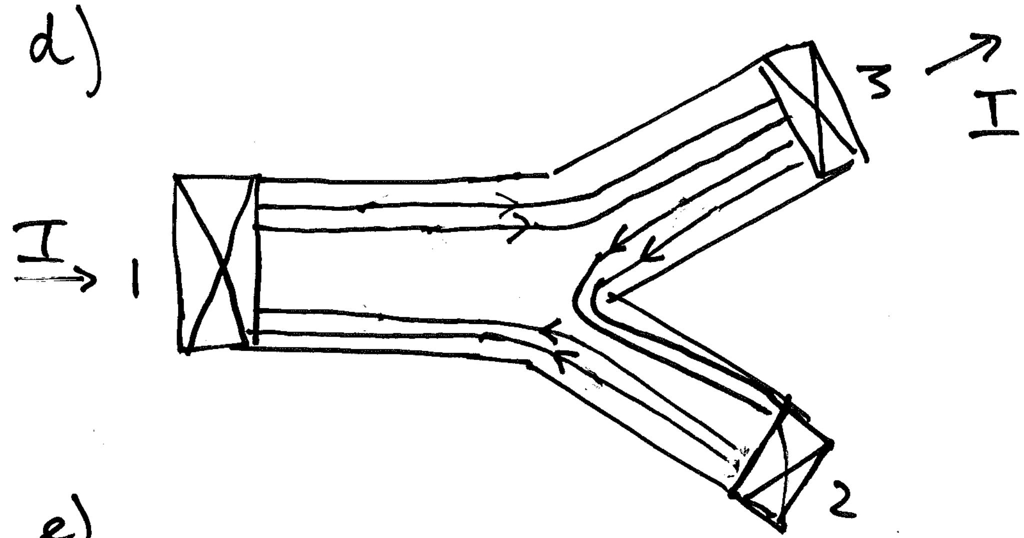
generates reflection at ends

$T < 1$

D

3 a, b, c see exam 17-6-2015

d)



e)

contacts 1 and 3 are current contacts
contact 2 is voltage contact.
→ adjusts the voltage such that
incoming current = outgoing current

$$\mu_3 \times \frac{2e^2}{h} \times 2 \qquad \mu_2 \times \frac{2e^2}{h} \times 2$$

Spin degeneracy = 2

therefore: $\mu_3 = \mu_2$ $\quad \mu_3 - \mu_1 = \frac{h}{4e^2} I$

voltage between 1 an 2 $= \frac{h}{4e^2} I$

f) linear system: current reverses
→ voltage reverses sign

g) yes : now $\mu_2 = \mu_1$ $\quad V_{12} = 0$

E

4 a) $I = I_c \sin\varphi$ $\qquad I_c = \frac{\pi}{2} \frac{\Delta}{e} G_t$

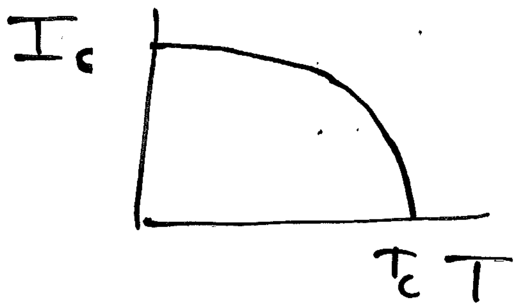
$V = \frac{\hbar}{2e} \frac{d\varphi}{dt}$

Δ: energy gap

G_t: normal state conductance

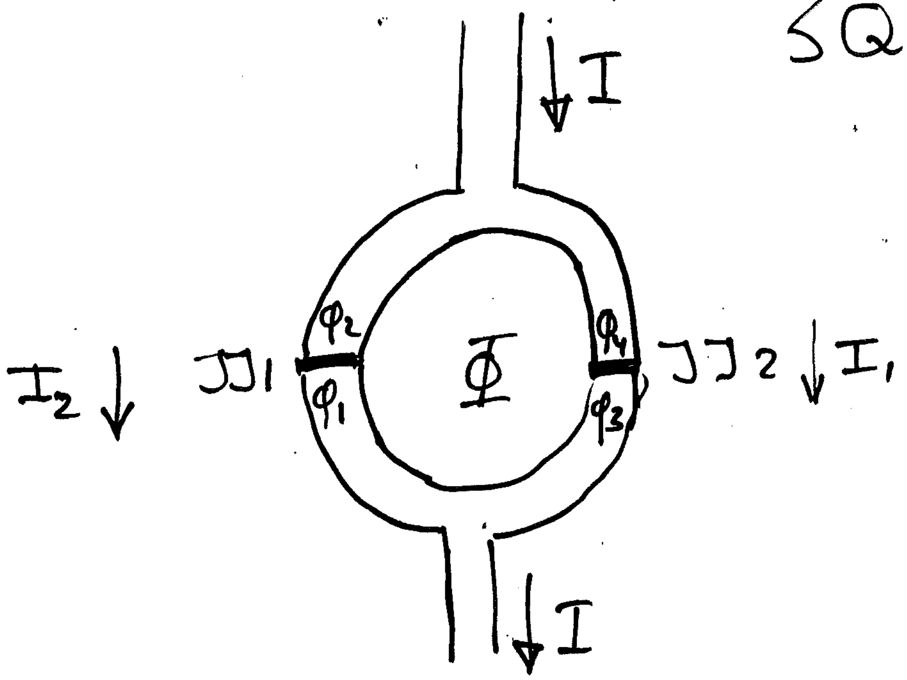
b) $V = \frac{\hbar}{2e} \frac{d\varphi}{dt}$ is exact

c)



d)

SQUID



(F)

$$I_1 = I_c \sin(\varphi_2 - \varphi_1)$$

$$I_2 = I_c \sin(\varphi_4 - \varphi_3 + \varphi_{AB}) \qquad \varphi_{AB} = 2\pi \frac{\Phi}{h/2e}$$

$$I = I_1 + I_2$$

assume $I_c \ll$ critical current of superconducting strips $\longrightarrow$

$\nabla\varphi$ in strips $\approx 0 \longrightarrow \varphi_1 = \varphi_3$ and $\varphi_2 = \varphi_4$

$$\Delta\varphi = \varphi_2 - \varphi_1 = \varphi_4 - \varphi_3$$

$$I = I_c \sin(\Delta\varphi) + I_c \sin\left(\Delta\varphi + 2\pi \frac{\Phi}{h/2e}\right)$$

$$= I_c \sin(\Delta\varphi) + I_c\left\{\sin(\Delta\varphi).\cos\left(2\pi\frac{\Phi}{h/2e}\right) + \cos(\Delta\varphi)\sin\left(2\pi\frac{\Phi}{h/2e}\right)\right\}$$

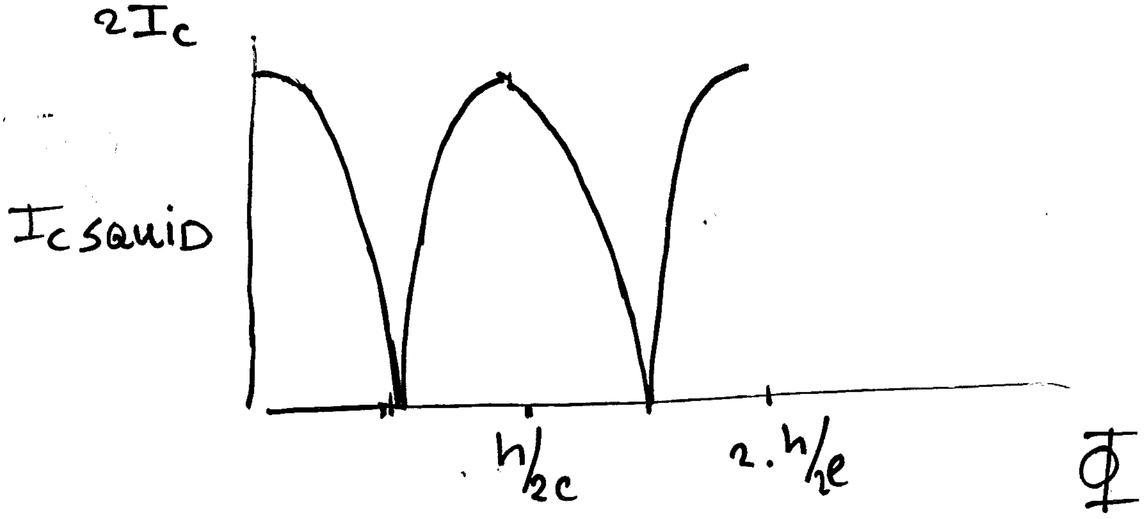
$$= I_c \sin(\Delta\varphi)\left\{1 + \cos\left(2\pi\frac{\Phi}{h/2e}\right)\right\} + I_c \cos(\Delta\varphi)\sin\left(2\pi\frac{\Phi}{h/2e}\right)$$

note : Φ is fixed $\quad \Delta\varphi$ is free parameter

$\Delta\varphi$ adjusts itself to maximize $I_{c\,SQUID}$

$$I_{c\,SQUID} = I_c\sqrt{\left\{1 + \cos\left(2\pi\frac{\Phi}{h/2e}\right)\right\}^2 + \sin^2\left(2\pi\frac{\Phi}{h/2e}\right)}$$

$$= I_c\sqrt{2 + 2\cos\left(2\pi\frac{\Phi}{h/2e}\right)}$$



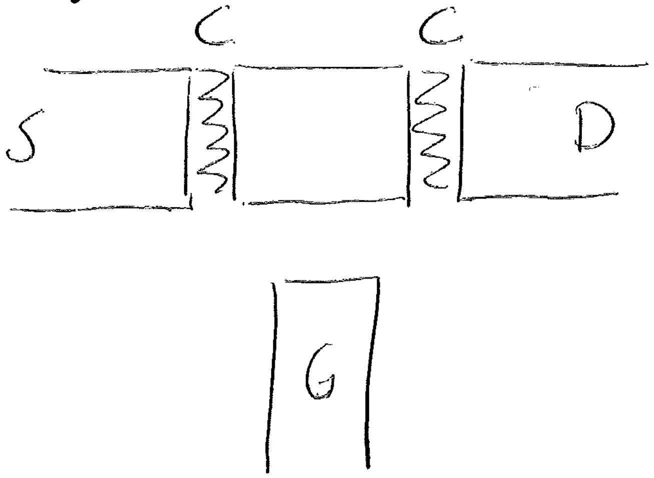
note : I_c squid $= 2I_c$ for $\Phi = 0,\ h/2e$ etc.

→ Supercurrents of JJ add.

I_c squid $= 0$ for $\Phi = h/4e,\ 3/4\ h/e$ etc

→ Supercurrents through JJ are always opposite → critical current $= 0$

5 a)



SET

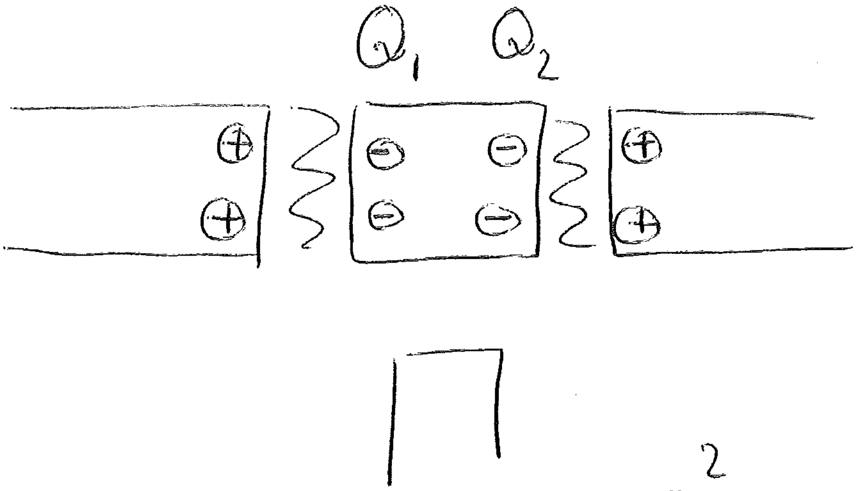
(H)

situation 0

neutral island

convention : ⊖ or ⊕ : 1/4 electroncharge

situation 1 : neutral island
after tunneling of 1 electron



Charging energy: $\frac{1}{2}\frac{Q_1^2}{C} + \frac{1}{2}\frac{Q_2^2}{C}$

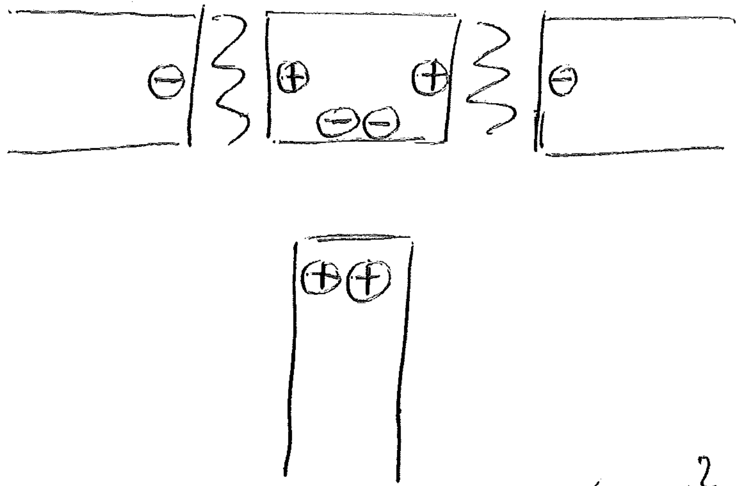
$$= \frac{1}{8}\frac{e^2}{C} + \frac{1}{8}\frac{e^2}{C} = \frac{1}{4}\frac{e^2}{C}$$

energy change :

$$\Delta E = \frac{1}{4}\frac{e^2}{C} - 0 = \frac{1}{4}\frac{e^2}{C} \rightarrow$$ Coulomb blockade

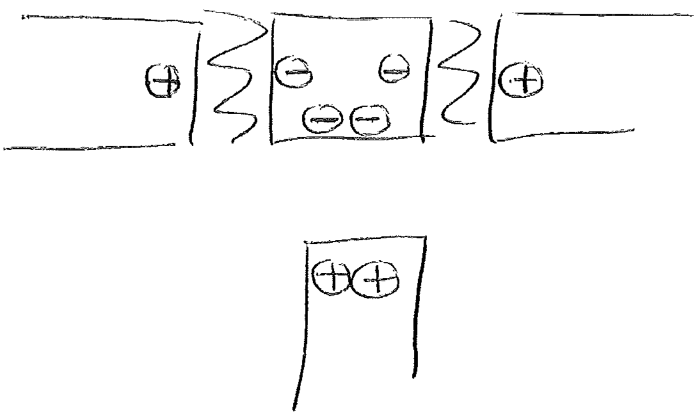
Situation 2 gate bias $= + \frac{1}{2} e$ (I)

a) before tunneling:



charging energy: $\frac{1}{2} \frac{\left(\frac{1}{4} e\right)^2}{C} + \frac{1}{2} \frac{\left(\frac{1}{4} e\right)^2}{C} = \frac{1}{16} \frac{e^2}{C}$

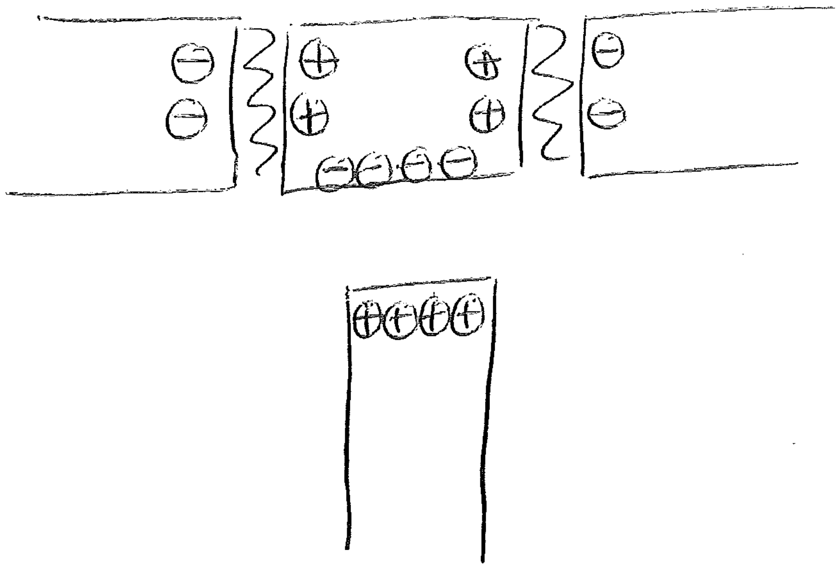
b) after tunneling



charging energy the same $\longrightarrow$ No Coulomb blockade

J

situation 3 gate bias = +1e



the system minimizes its charging energy by the tunneling of an electron

→ back to situation 0

5 b)

(K)

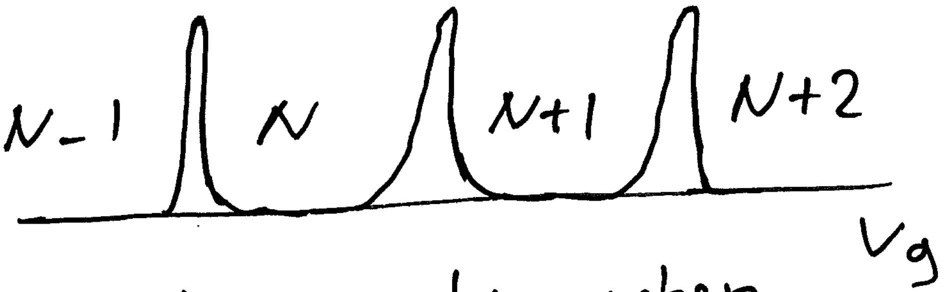
$kT < \Delta E \qquad \Delta E \approx \frac{e^2}{C}$

$V_{DS} < \frac{\Delta E}{e} \qquad V_{DS}$: source-drain voltage

$R > e^2/h$ tunnel resistance has to be larger than typically e^2/h

5 c) see above

5 d)



SET conductance peaks when charging energy for N and N+1 electrons on the island is the same

at low T Coulomb blockade becomes already effective for small deviations from the above condition $\longrightarrow$ sharp conductance peaks as function of V_g

$\longrightarrow$ SET can measure very small ($<10^{-4}e$) changes in charge environment.